



Option Pricing Basics

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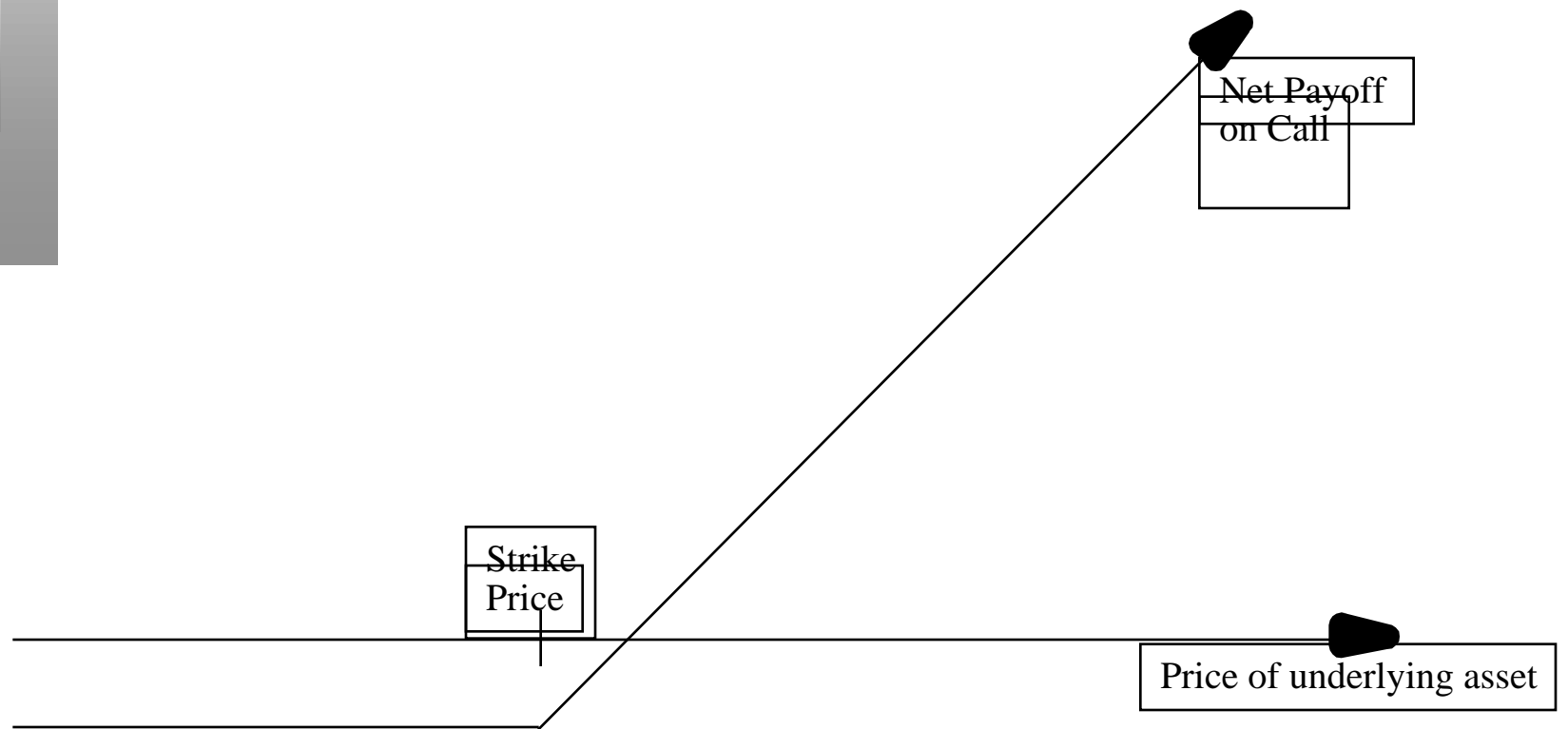
What is an option?

- An option provides the holder with the **right** to buy or sell a specified quantity of an underlying asset at a fixed price (called a strike price or an exercise price) at or before the expiration date of the option.
- Since it is a right and **not an obligation**, the holder can choose not to exercise the right and allow the option to expire.
- There are two types of options - **call** options (right to buy) and **put** options (right to sell).

Call Options

- A call option gives the buyer of the option the right to buy the underlying asset at a fixed price (strike price or K) at any time prior to the expiration date of the option. The buyer pays a price for this right.
- At expiration,
 - If the value of the underlying asset (S) $>$ Strike Price(K)
 - Buyer makes the difference: $S - K$
 - If the value of the underlying asset (S) $<$ Strike Price (K)
 - Buyer does not exercise
- More generally,
 - the value of a call increases as the value of the underlying asset increases
 - the value of a call decreases as the value of the underlying asset decreases

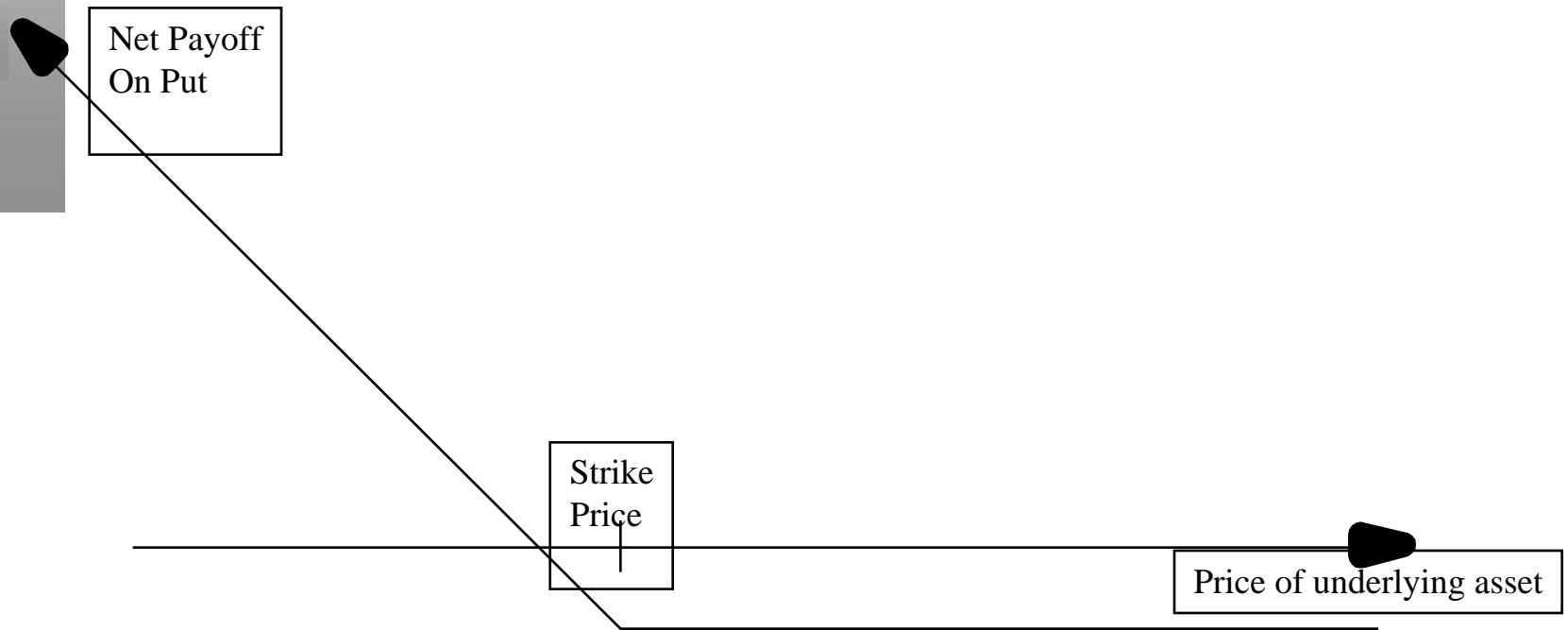
Payoff Diagram on a Call



Put Options

- A put option gives the buyer of the option the right to sell the underlying asset at a fixed price at any time prior to the expiration date of the option. The buyer pays a price for this right.
- At expiration,
 - If the value of the underlying asset (S) < Strike Price (K)
 - Buyer makes the difference: $K-S$
 - If the value of the underlying asset (S) > Strike Price (K)
 - Buyer does not exercise
- More generally,
 - the value of a put decreases as the value of the underlying asset increases
 - the value of a put increases as the value of the underlying asset decreases

Payoff Diagram on Put Option



Determinants of option value

- Variables Relating to Underlying Asset
 - Value of Underlying Asset; as this value increases, the right to buy at a fixed price (calls) will become more valuable and the right to sell at a fixed price (puts) will become less valuable.
 - Variance in that value; as the variance increases, both calls and puts will become more valuable because all options have limited downside and depend upon price volatility for upside.
 - Expected dividends on the asset, which are likely to reduce the price appreciation component of the asset, reducing the value of calls and increasing the value of puts.
- Variables Relating to Option
 - Strike Price of Options; the right to buy (sell) at a fixed price becomes more (less) valuable at a lower price.
 - Life of the Option; both calls and puts benefit from a longer life.
- Level of Interest Rates; as rates increase, the right to buy (sell) at a fixed price in the future becomes more (less) valuable.

American versus European options: Variables relating to early exercise

- An American option **can be exercised at any time prior** to its expiration, while a European option can be exercised only at expiration.
 - The possibility of early exercise makes **American options more valuable** than otherwise similar European options.
 - However, in most cases, the **time premium** associated with the remaining life of an option makes **early exercise sub-optimal**.
- While early exercise is generally not optimal, there are two exceptions:
 - One is where the underlying asset **pays large dividends**, thus reducing the value of the asset, and of call options on it. In these cases, call options may be exercised just before an ex-dividend date, if the time premium on the options is less than the expected decline in asset value.
 - The other is when an investor holds both the underlying asset and deep in-the-money puts on that asset, at a time when **interest rates are high**. The time premium on the put may be less than the potential gain from exercising the put early and earning interest on the exercise price.

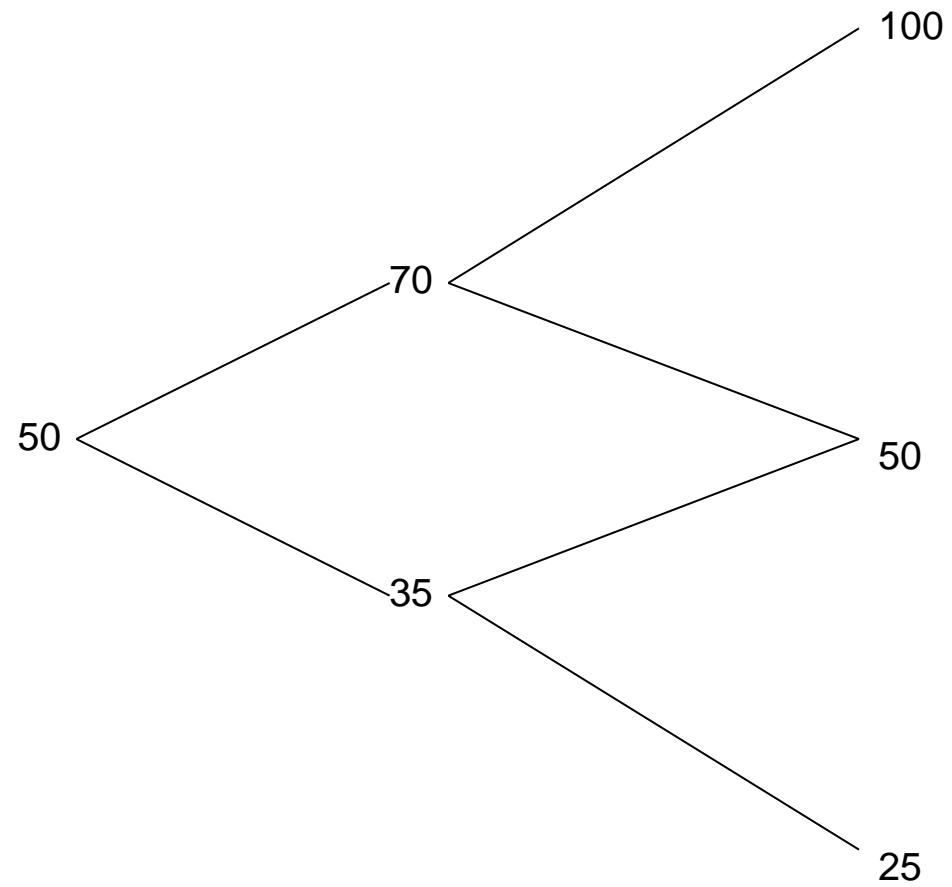
A Summary of the Determinants of Option Value

<i>Factor</i>	<i>Call Value</i>	<i>Put Value</i>
Increase in Stock Price	Increases	Decreases
Increase in Strike Price	Decreases	Increases
Increase in variance of underlying asset	Increases	Increases
Increase in time to expiration	Increases	Increases
Increase in interest rates	Increases	Decreases
Increase in dividends paid	Decreases	Increases

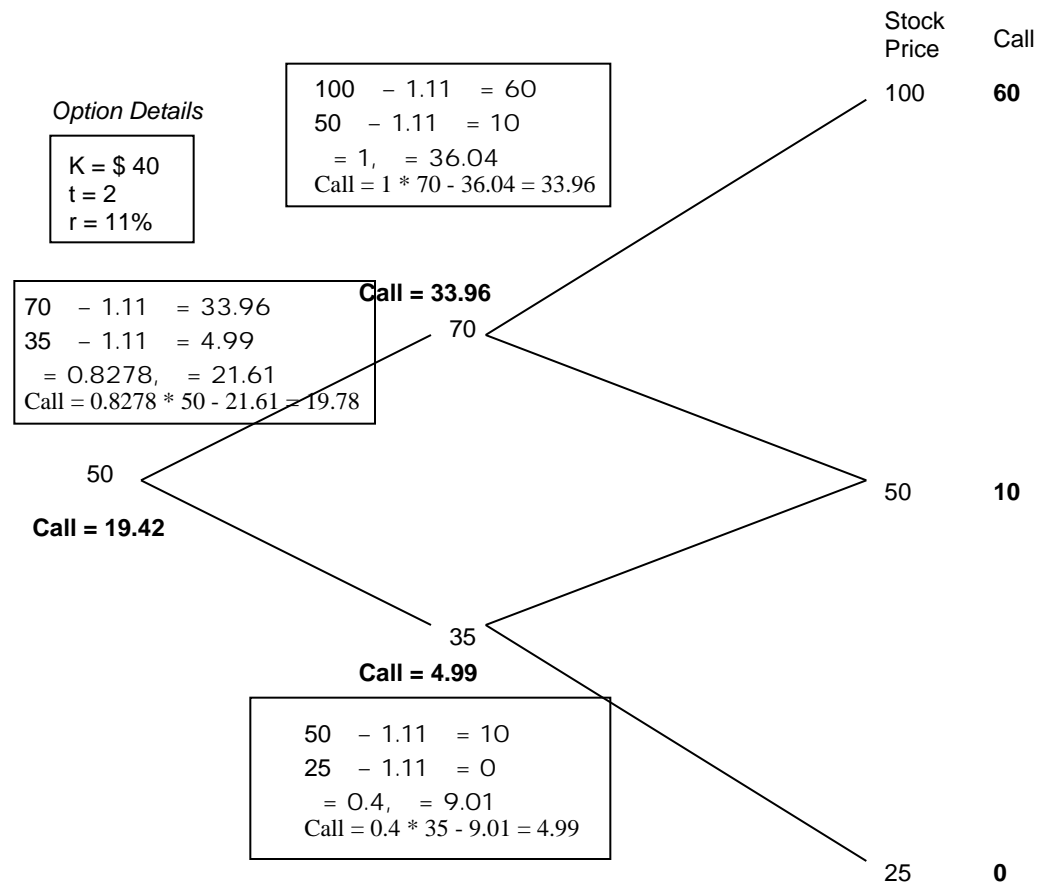
Creating a replicating portfolio

- The objective in creating a replicating portfolio is to use a combination of riskfree borrowing/lending and the underlying asset to create the same cashflows as the option being valued.
 - Call = Borrowing + Buying of the Underlying Stock
 - Put = Selling Short on Underlying Asset + Lending
 - The number of shares bought or sold is called the **option delta**.
- The principles of arbitrage then apply, and the value of the option has to be equal to the value of the replicating portfolio.

The Binomial Model



The Replicating Portfolio



The Limiting Distributions....

- As the time interval is shortened, the limiting distribution, as $t \rightarrow 0$, can take one of two forms.
 - If as $t \rightarrow 0$, **price changes become smaller**, the limiting distribution is the normal distribution and the **price process is a continuous one**.
 - If as $t \rightarrow 0$, **price changes remain large**, the limiting distribution is the poisson distribution, i.e., a **distribution that allows for price jumps**.
- **The Black-Scholes model** applies when the **limiting distribution is the normal distribution**, and explicitly assumes that the price process is continuous and that there are no jumps in asset prices.

The Black-Scholes Model

- The version of the model presented by Black and Scholes was designed to value European options, which were dividend-protected.
- The value of a call option in the Black-Scholes model can be written as a function of the following variables:

S = Current value of the underlying asset

K = Strike price of the option

t = Life to expiration of the option

r = Riskless interest rate corresponding to the life of the option

σ^2 = Variance in the $\ln(\text{value})$ of the underlying asset

The Black Scholes Model

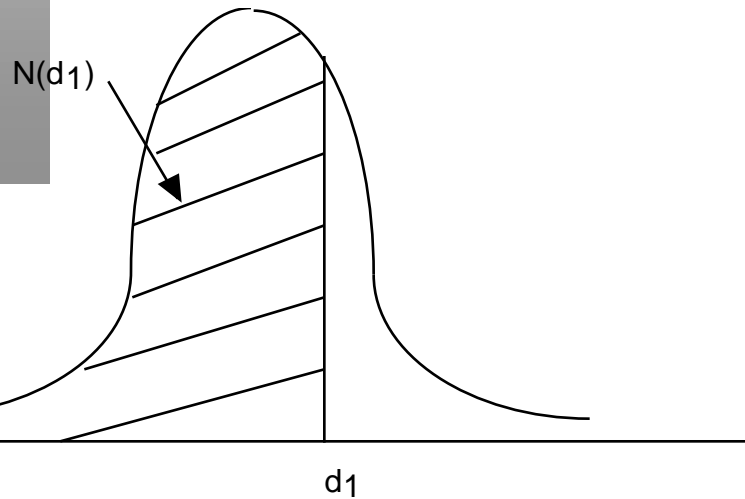
Value of call = $S N(d_1) - K e^{-rt} N(d_2)$

where,

$$d_1 = \frac{\ln \frac{S}{K} + (r + \frac{\sigma^2}{2}) t}{\sigma \sqrt{t}}$$

- $d_2 = d_1 - \sigma \sqrt{t}$
- The replicating portfolio is embedded in the Black-Scholes model. To replicate this call, you would need to
 - Buy $N(d_1)$ shares of stock; $N(d_1)$ is called the option delta
 - Borrow $K e^{-rt} N(d_2)$

The Normal Distribution



d	$N(d)$	d	$N(d)$	d	$N(d)$
-3.00	0.0013	-1.00	0.1587	1.05	0.8531
-2.95	0.0016	-0.95	0.1711	1.10	0.8643
-2.90	0.0019	-0.90	0.1841	1.15	0.8749
-2.85	0.0022	-0.85	0.1977	1.20	0.8849
-2.80	0.0026	-0.80	0.2119	1.25	0.8944
-2.75	0.0030	-0.75	0.2266	1.30	0.9032
-2.70	0.0035	-0.70	0.2420	1.35	0.9115
-2.65	0.0040	-0.65	0.2578	1.40	0.9192
-2.60	0.0047	-0.60	0.2743	1.45	0.9265
-2.55	0.0054	-0.55	0.2912	1.50	0.9332
-2.50	0.0062	-0.50	0.3085	1.55	0.9394
-2.45	0.0071	-0.45	0.3264	1.60	0.9452
-2.40	0.0082	-0.40	0.3446	1.65	0.9505
-2.35	0.0094	-0.35	0.3632	1.70	0.9554
-2.30	0.0107	-0.30	0.3821	1.75	0.9599
-2.25	0.0122	-0.25	0.4013	1.80	0.9641
-2.20	0.0139	-0.20	0.4207	1.85	0.9678
-2.15	0.0158	-0.15	0.4404	1.90	0.9713
-2.10	0.0179	-0.10	0.4602	1.95	0.9744
-2.05	0.0202	-0.05	0.4801	2.00	0.9772
-2.00	0.0228	0.00	0.5000	2.05	0.9798
-1.95	0.0256	0.05	0.5199	2.10	0.9821
-1.90	0.0287	0.10	0.5398	2.15	0.9842
-1.85	0.0322	0.15	0.5596	2.20	0.9861
-1.80	0.0359	0.20	0.5793	2.25	0.9878
-1.75	0.0401	0.25	0.5987	2.30	0.9893
-1.70	0.0446	0.30	0.6179	2.35	0.9906
-1.65	0.0495	0.35	0.6368	2.40	0.9918
-1.60	0.0548	0.40	0.6554	2.45	0.9929
-1.55	0.0606	0.45	0.6736	2.50	0.9938
-1.50	0.0668	0.50	0.6915	2.55	0.9946
-1.45	0.0735	0.55	0.7088	2.60	0.9953
-1.40	0.0808	0.60	0.7257	2.65	0.9960
-1.35	0.0885	0.65	0.7422	2.70	0.9965
-1.30	0.0968	0.70	0.7580	2.75	0.9970
-1.25	0.1056	0.75	0.7734	2.80	0.9974
-1.20	0.1151	0.80	0.7881	2.85	0.9978
-1.15	0.1251	0.85	0.8023	2.90	0.9981
-1.10	0.1357	0.90	0.8159	2.95	0.9984
-1.05	0.1469	0.95	0.8289	3.00	0.9987
-1.00	0.1587	1.00	0.8413		

Adjusting for Dividends

- If the dividend yield ($y = \text{dividends} / \text{Current value of the asset}$) of the underlying asset is expected to remain unchanged during the life of the option, the Black-Scholes model can be modified to take dividends into account.

$$C = S e^{-yt} N(d_1) - K e^{-rt} N(d_2)$$

$$d_1 = \frac{\ln \frac{S}{K} + (r - y + \frac{\sigma^2}{2}) t}{\sigma \sqrt{t}}$$

$$d_2 = d_1 - \sigma \sqrt{t}$$

- The value of a put can also be derived:

$$P = K e^{-rt} (1 - N(d_2)) - S e^{-yt} (1 - N(d_1))$$